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DETERMINATION OF SIZE OF STABILIZING FINS FOR SMALL WATERPLANE AREA, TWIN-HULL SHIPS

C. M. Lee, et al

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A theoretical approach for determining the size of stabilizing fins desirable for SWATH 4A and SWATH 4B models is described. Determination of fin size is made on the basis of retaining pitch stability for high speeds as well as augmenting the heave and pitch damping for motion in waves.

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NOTATION

A	Wave amplitude
A_{ij}	Added mass coefficient in the ith mode due to motion in the jth mode
A_{wp}	Waterplane area at load waterline
a_{33}	Sectional heave added mass
B_{ij}	Damping coefficient in the ith mode due to the motion in the jth mode
B^*_{33}	Vertical force rate due to heave velocity
B^*_{35}	Vertical force rate due to pitch angular velocity
B^*_{55}	Pitch moment rate due to pitch angular velocity
B^*_{53}	Pitch moment rate due to heave velocity
$\overline{BG}$	Vertical distance between the center of buoyancy and the center of gravity of ship
b_{33}	Sectional heave damping coefficient
C_{ij}	Restoring coefficient in the ith mode due to the motion of the jth mode
$C_{L\alpha i}$	Lift-curve slope of the ith fin
c_i	Chord of the ith fin
F'_3, F'_5	Wave exciting coefficients as defined below Equations (14)
$\overline{GM}_\ell$	Longitudinal metacentric height
g	Gravitational acceleration
I_5	Mass moment of inertia of ship about the y axis
L	Overall ship length
L_1	Length between Stations 0 and 20
ℓ_i	The x coordinate of the 1/4-chord point of the ith fin
M	Mass of ship
M'_q	Coefficient of pitch moment due to pitch angular velocity
M'_w	Coefficient of pitch moment due to heave velocity
M_{wp}	Moment of waterplane area about the y axis

(0, x, y, z)	Right-handed Cartesian coordinate system; see definition on page 2
s_i	Span of the ith fin
U	Ship speed
Z'_q	Coefficient of heave force due to pitch angular velocity
Z'_w	Coefficient of heave force due to heave velocity
Δ	Displacement of ship
ϵ_1	Argument of $(i\omega - \lambda_1)(i\omega - \lambda_2)$
ϵ_2	Argument of $(i\omega - \lambda_3)(i\omega - \lambda_4)$
ζ_i	Damping ratio, i = 3 for heave and i = 5 for pitch
λ_n	$(=\lambda_{nR} + i\,\lambda_{nI})$ nth stability root
ξ_3	Heave displacement, positive upward
ξ_5	Pitch angular displacement, positive bow down
ρ	Mass density of water
ω	Wave encounter frequency in radian per unit time
ω_{0H}	Undamped natural frequency for heave mode
ω_{0P}	Undamped natural frequency for pitch mode

ABSTRACT

A theoretical approach for determining the size of stabilizing fins desirable for SWATH 4A and SWATH 4B models is described. Determination of fin size is made on the basis of retaining pitch stability for high speeds as well as augmenting the heave and pitch damping for motion in waves.

ADMINISTRATIVE INFORMATION

This investigation has been authorized and funded by the Naval Material Command under the Small Waterplane Area, Twin-Hull Program, Work Unit 1-1170-026.

INTRODUCTION

Small waterplane area, twin-hull (SWATH) ships have larger natural periods for heave and pitch modes than monohull ships of equivalent displacement because of the small waterplane area. The larger natural period, together with smaller wave-excitation force due to the submerged hulls, provides SWATH ships with a seakeeping advantage in moderate seas. The smallness of the waterplane area, however, can result in pitch instability when a SWATH ship cruises with speeds higher than a certain limit. The cause of pitch-mode instability mainly stems from the so-called Munk moment, which is proportional to the square of the speed and provides a destabilizing pitch moment. For the SWATH 4 model it was predicted that this instability would occur at approximately 27 knots.

It was therefore necessary to design a pair of stabilizing fins that would provide adequate stability over the entire range from 0 to 32 knots. This design was determined on the basis of coupled pitch and heave stability equations since surge was not considered to be important. One fin was installed on the inboard side of each hull of SWATH 4 at approximately the 0.84 L position. Free-running model experiments in waves were carried out both with and without the stabilizing fins. The results of model experiments confirmed that pitch instability did indeed occur when the full-scale speed exceeded 20 knots, and the use of fins resulted in a significant improvement in the overall performance of the SWATH 4 configuration throughout the speed range of interest.

The encouraging results of the SWATH 4 experiments led to the present study to determine adequate stabilizing fin sizes for two new SWATH forms which were provided by the SWATH Program Office at the Center. The two SWATH forms are designated as SWATH 4A and SWATH 4B. The SWATH 4A is a geosim of SWATH 4 with a reduced-scale ratio of 0.833 to 1, while SWATH 4B has the same displacement as SWATH 4A but greater overall length, a smaller hull diameter, a greater strut length, and a smaller strut beam. The SWATH 4B has a longitudinal metacentric height ($\overline{GM}_\ell$) about twice as large as SWATH 4A.

A SWATH ship with two separated struts at fore and aft can provide a large $\overline{GM_\ell}$. It is of interest to learn the effects of increased $\overline{GM_\ell}$ on the vertical plane stability as well as on the vertical motion in waves in the vicinity of resonant frequencies. The reason for choosing SWATH 4B is intended to pursue the aforesaid interest. Figure 1 shows the profiles of SWATH 4A and SWATH 4B, and Table 1 shows the principal characteristics.

The analysis for determining adequate fin sizes is based on the coupled heave and pitch equations for stability and will be described in detail in the following sections. The size of the fins is determined by the following three conditions. The fins must provide:

1. pitch stability up to 40 knots in calm water,
2. increased damping for the heave and pitch modes, and
3. reasonable natural periods for heave and pitch.

DESCRIPTION OF ANALYSIS

Assume that a SWATH ship translating with a forward speed U is momentarily disturbed so as to induce a coupled heave and pitch motion. Our objective is to study whether the ship at a given speed has sufficient stability to restore itself to its original equilibrium in a reasonable time. If the ship is found unstable, a stabilizing fin or fins will be introduced in the analysis.

The coupled heave and pitch equations of motion without excitation can be expressed in the form

$$(M + A_{33})\ddot{\xi}_3 + B_{33}\dot{\xi}_3 + C_{33}\xi_3 + A_{35}\ddot{\xi}_5 + B_{35}\dot{\xi}_5 + C_{35}\xi_5 = 0 \tag{1}$$

$$A_{53}\ddot{\xi}_3 + B_{53}\dot{\xi}_3 + C_{53}\xi_3 + (I_5 + A_{55})\ddot{\xi}_5 + B_{55}\dot{\xi}_5 + C_{55}\xi_5 = 0 \tag{2}$$

Equations (1) and (2) are formulated with respect to the body coordinate system at the equilibrium position of the ship, i.e., no motion except a steady forward translation. The origin of the coordinate system 0 is located on the calm water free surface and, together with the center of gravity, lies in the longitudinal plane of symmetry of the two hulls. The axes, 0x, 0y, and 0z are respectively directed toward bow, port, and vertically upward. $\xi_3(t)$ is the heave displacement, which is positive upward, and $\xi_5(t)$ is the pitch angular displacement about the y-axis, which is positive bow down. M is the mass of the ship; I_5 is the mass moment of inertia about the y-axis; A_{ij}, B_{ij}, and C_{ij} for i,j = 3 and 5 are, respectively, the added inertia, damping, and restoring coefficients.

TABLE 1 HULL CHARACTERISTICS OF SWATH 4A AND SWATH 4B

Description	Unit	SWATH 4A	SWATH 4B
Displacement	Long tons	2379	2379
Displacement Main Hull	Long tons	1852	1829
Length Overall	Feet	239 7	275 9
Length of Strut	Feet	188 9	229 54
Maximum Strut Thickness	Feet	6 67	5 147
Maximum Hull Diameter	Feet	15 0	13 7
Draft	Feet	25 0	22 63
Waterplane Area	$Feet^2$	1893	1925
Waterplane Area Moment of Inertia	$Feet^4$	$3\,645 \times 10^6$	5.997×10^6
Center of Buoyancy from Nose of Hull	Feet	115 6	133.92
Center of Flotation From Nose of Hull	Feet	117 7	135 93
$\overline{BG}$	Feet	15 19	14.06
Longitudinal $\overline{GM}$	Feet	27.7	58 4
Radius of Gyration for Pitch	Feet	52 1	61.8

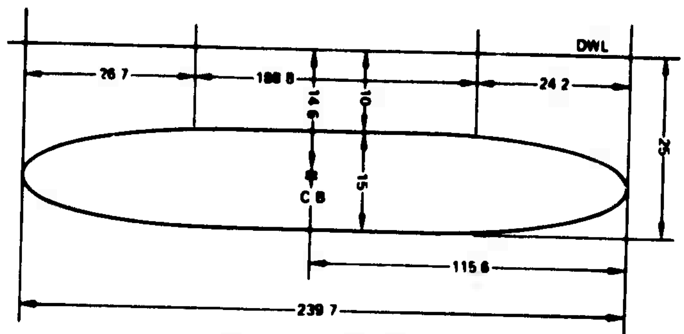

Figure 1a – SWATH 4A

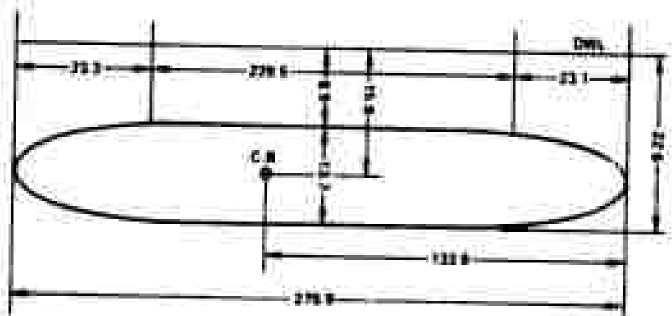

Figure 1b – SWATH 4B

Figure 1 – Profiles of SWATH 4A and SWATH 4B

(Numbers are in feet, and drawings are not to scale)

It will be assumed that the hydrodynamic coefficients A_{ij} and B_{ij} are made up of three contributing factors:

1. irrotational flow with the free-surface boundary disturbed by the body without the fin,
2. real fluid effect on the bare hull, and
3. fin effect.

Under this assumption the hydrodynamic and restoring coefficients are given by

$$A_{33} = \int_L a_{33}(x)dx \tag{3.1}$$

$$B_{33} = \int b_{33}(x)dx + UB^*_{33} + \sum_{i=1}^{N} \frac{1}{2}\rho U c_i s_i C_{L\alpha i} \tag{3.2}$$

$$C_{33} = \rho g A_{wp} + C_{33}^{(F)} \tag{3.3}$$

$$A_{35} = -\int xa_{33}(x)dx \tag{3.4}$$

$$B_{35} = -\int xb_{33}(x)dx + U(A_{33} + B^*_{35}) - \sum_{i=1}^{N} \frac{1}{2}\rho U c_i s_i \ell_i C_{L\alpha i} \tag{3.5}$$

$$C_{35} = -\rho g M_{wp} + UB_{33} + C_{35}^{(F)} \tag{3.6}$$

$$A_{53} = -\int xa_{33}(x)dx \tag{3.7}$$

$$B_{53} = -\int xb_{33}(x)dx + UB^*_{53} - \sum_{i=1}^{N} \frac{1}{2}\rho U c_i s_i \ell_i C_{L\alpha i} \tag{3.8}$$

$$C_{53} = -\rho g M_{wp} + C_{53}^{(F)} \tag{3.9}$$

$$A_{55} = \int x^2 a_{33}(x)dx \tag{3.10}$$

$$B_{55} = \int x^2 b_{33}(x)dx + UB^*_{55} + \sum_{i=1}^{N} \frac{1}{2} \rho U c_i s_i \ell_i^2 C_{L\alpha i} \tag{3.11}$$

$$C_{55} = \Delta \overline{GM}_\ell + U^2 C_{55}^{(M)} - \sum_{i=1}^{N} \frac{1}{2} \rho U^2 c_i s_i \ell_i C_{L\alpha i} + C_{55}^{(I)} \tag{3.12}$$

where $\int_L$ = integration over the ship length

$a_{33}(x)$ = sectional added mass of cross section at x

$b_{33}(x)$ = sectional damping due to wavemaking of cross section at x

B^*_{33} = vertical force rate due to heave velocity*

N = total number of fins

ρ = mass density of water

c_i = chord of the ith fin

s_i = span of the ith fin

$C_{L\alpha i}$ = lift-curve slope of the ith fin

g = gravitational acceleration

A_{wp} = waterplane area

B^*_{35} = vertical force rate due to pitch angular velocity*

ℓ_i = x coordinate of the ith fin

M_{wp} = waterplane area moment about the y axis

B^*_{53} = pitch moment rate due to heave velocity

B^*_{55} = pitch moment rate due to pitch angular velocity

Δ = displacement of ship

$\overline{GM}_\ell$ = longitudinal metacentric height

$C_{55}^{(M)}$ = Munk moment = B^*_{53}

The coefficient $C_{ij}^{(I)}$ for i, j = 3 and 5 are to account for the effects of sinkage and trim resulting from forward speed of the ship in the free surface on the heave force and pitch

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*Contribution of bare hull only and refers to nonwavemaking part.

moment derivatives. The effect of these coefficients on the stability analysis were assumed to be negligible. The importance of these terms increases as the draft decreases.

The coefficients with an asterisk represent the contributions of the hull without fins to the force and moment not attributable to wavemaking. Since these quantities are strongly influenced by viscous effects, it was necessary to estimate them on the basis of available experimental data from experiments with submarine models. The viscous effects thus obtained ignore the presence of the struts and also the hydrodynamic interference effect from the other hull. If we use the notation employed in the equations of motion for submarines,[1] which are often called submarine derivatives, then we may write

$$
\begin{aligned}
B^*_{33} &= -\frac{1}{2}\rho L^2 Z'_w \\
B^*_{35} &= -\frac{1}{2}\rho L^3 Z'_q \\
B^*_{53} &= -\frac{1}{2}\rho L^3 M'_w \\
B^*_{55} &= -\frac{1}{2}\rho L^4 M'_q
\end{aligned}
\qquad (3.13)
$$

where L is the overall length of the ship.

Although it is known that several of the coefficients of Equations (1) and (2) are frequency dependent, they have been treated as constants, and the values selected were those associated with frequencies close to the natural heave frequency. It was felt that this was a reasonable approximation for the following reasons.

1. Since SWATH vehicles are more deeply submerged than surface craft, the frequency dependence of the coefficients are significantly smaller; especially the frequency range about the natural heave frequency.

2. For SWATH vehicles of this study, the oscillatory response to disturbances is dominated by motions at a single frequency, viz., the heave natural frequency. The added mass terms vary only mildly with frequency here. Furthermore, these terms represent only about 40 percent of the virtual mass.

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[1] Gertler, M. and G. R. Hagen, "Standard Equations for Motion for Submarine Simulation," NSRDC Report 2510 (1967).

3. The purpose of the study was to investigate the relative merits of various stabilizing fin sizes in providing stability and adequate damping of the motions. The effects of these fins were generally much larger than the effects due to the terms that would tend to vary most with frequency, viz., the damping terms due to wavemaking.

4. Comparisons of theory with data for most of these frequency-dependent damping coefficients often show differences of the same order of magnitude as the variations with frequency.

Equations (1) and (2) are a pair of linear, homogeneous differential equations with constant coefficients. Solutions of such equations are well known[2] and are given by the real part of

$$
\begin{aligned}
\xi_3(t) &= \sum_{n=1}^{N} a_n \, e^{\lambda_n t} \\
\xi_5(t) &= \sum_{n=1}^{N} b_n \, e^{\lambda_n t}
\end{aligned}
\qquad (4)
$$

where a_n and b_n are arbitrary constants that depend on the initial conditions, and the λ_n are the roots of the characteristic equations. The characteristic equation may be obtained by substituting any pair of the solutions in Equation (4) into Equations (1) and (2). Thus, if we substitute the solutions

$$
\begin{aligned}
\xi_3(t) &= a_n \, e^{\lambda_n t} \\
\xi_5(t) &= b_n \, e^{\lambda_n t}
\end{aligned}
\qquad (5)
$$

into Equations (1) and (2), we obtain

$$
\left\{(M + A_{33})\lambda^2 + B_{33}\lambda + C_{33}\right\} a_n + (A_{35}\lambda^2 + B_{35}\lambda + C_{35})b_n = 0
$$

$$
(A_{53}\lambda^2 + B_{53}\lambda + C_{53})a_n + \left\{(I_5 + A_{55})\lambda^2 + B_{55}\lambda + C_{55}\right\} b_n = 0
$$

[2] Hildebrand, F. B., "Advanced Calculus for Engineers," Prentice Hall, Inc., Englewood Cliffs, N. J. (1949).

To obtain solutions for a_n and b_n from simultaneous homogeneous equations as shown, the determinant of the equations should vanish. That is,

$$a\lambda^4 + b\lambda^3 + c\lambda^2 + d\lambda + e = 0 \tag{6}$$

or, on dividing through by a

$$\lambda^4 + b'\lambda^3 + c'\lambda^2 + d'\lambda + e' = 0$$

where

$$a = (M + A_{33})(I_5 + A_{55}) - A_{53}A_{35} \tag{7.1}$$

$$b = (M + A_{33})B_{55} - A_{53}B_{35} + B_{33}(I_5 + A_{55}) - B_{35}A_{35} \tag{7.2}$$

$$c = (M + A_{33})C_{55} - A_{53}C_{35} + B_{33}B_{55} - B_{53}B_{35} + (I_{55} + A_{55})C_{33} - A_{35}C_{53} \tag{7.3}$$

$$d = B_{33}C_{55} - B_{53}C_{35} + C_{33}B_{55} - C_{35}B_{53} \tag{7.4}$$

$$e = C_{33}C_{55} - C_{35}C_{53} \tag{7.5}$$

It is clear from Equation (6) that there are four roots. These roots can be obtained either algebraically[3] or on a computer, using a subroutine which provides the roots of polynomial equations. The closed form expression for the four roots of Equation (6) can be written as

$$\begin{pmatrix} \lambda_1 \\ \lambda_2 \end{pmatrix} = -\frac{b'}{4} + \frac{m}{2} (\pm) \frac{1}{2} \left[\left(m - \frac{b'}{2} \right)^2 - 2(t - n) \right]^{1/2} \tag{8.1}$$

$$\begin{pmatrix} \lambda_3 \\ \lambda_4 \end{pmatrix} = -\frac{b'}{4} - \frac{m}{2} (\pm) \frac{1}{2} \left[\left(m + \frac{b'}{2} \right)^2 - 2(t + n) \right]^{1/2} \tag{8.2}$$

[3]Dickson, L. E., "Theory of Equations," John Wiley and Sons, Inc., New York (1947).

$$\text{where } m = \left[\frac{b'^2}{4} - c' + t\right]^{1/2}$$

$$n = \frac{1}{2m}\left(\frac{b'}{2}\, t - d'\right)$$

$$t = \left[-\frac{q}{2} + \left\{\left(\frac{p}{3}\right)^3 + \left(\frac{q}{2}\right)^2\right\}^{1/2}\right]^{1/3} + \left[\frac{q}{2} - \left\{\left(\frac{p}{3}\right)^3 + \left(\frac{q}{2}\right)^2\right\}^{1/2}\right]^{1/3}$$

$$p = b'd' - 4e' - \frac{c'^2}{3}$$

$$q = -b'^2e' + 4c'e' - d'^2 + \frac{c'}{3}(b'd' - 4e') - \frac{2}{27}e'^3$$

As can be seen from Equations (8), the value of λ_n may be real or complex, with the complex value appearing in conjugate pairs. The motion corresponding to each real λ_n or each complex pair is called a normal mode. There are four possible kinds of normal modes of motion according to whether λ_n is real or complex* and whether it has a positive or negative real part. As can be inferred from Equation (4a) in the footnote the motion is

1. divergent, if λ_n is real and positive;
2. divergent oscillatory, if λ_n is complex and has a positive real part;
3. convergent, if λ_n is real and negative; and
4. convergent (or damped) oscillatory, if λ_n is complex and has a negative real part.

Modes 1 and 2 are unstable modes since, as can be seen, the amplitude of the motion grows with the time as long as the system remains linear. Modes 3 and 4 are stable modes since amplitude of the motion decays with time to zero.

The necessary and sufficient condition for stability, i.e., that all of the roots of Equation (6) have negative real parts is given by Routh's stability criteria[4]

*If, for example, the roots for one of the modes were given by $\lambda_1 = \lambda_{aR} + i\lambda_{aI}$ and $\lambda_2 = \lambda_{aR} - i\lambda_{aI}$, then by substituting these into Equation (4) with N = 2, and taking the real part, we would obtain the following typical form of solution for this mode

$$\xi_3(t) = e^{\lambda_{aR}t}(c_a \cos \lambda_{aI}t + d_a \sin \lambda_{aI}t) \tag{4a}$$

where λ_{aI} is the circular oscillation frequency, and c_a and d_a are real constants.

[4]Routh, E. J., "Dynamics of a System of Rigid Bodies," 6th Edition, The Macmillan Company, London (1905).

$$a, b, d, e > 0$$

$$bcd - ad^2 - b^2e > 0 \tag{9}$$

Numerical values of primary interest, which are related to λ_n, are the period of oscillation T_n, the damping ratio ζ_n, and the time to halve or double the amplitude of the disturbed motion.

If the modes are oscillatory, the natural period are given by

$$T_1 = 2\pi/|\lambda_{1I}| \tag{10.1}$$

$$T_3 = 2\pi/|\lambda_{3I}| \tag{10.2}$$

if we write λ_n as follows

$$\lambda_n = \lambda_{nR} \pm i\lambda_{nI}$$

According to Equation (4), the disturbed motion, in general, will involve both modes so that each heave and pitch motion will contain all the oscillatory components present. However, if the coupling between the heave and pitch motions is weak, i.e., if the contributions of the A_{ij}, B_{ij}, and C_{ij} $(i \neq j)$ terms in Equations (1) and (2) have small effect on the coefficients of the characteristic Equation (6), then useful insights can be obtained by both examining the uncoupled heave and pitch characteristics and comparing them with the corresponding coupled values. Furthermore, for the present case, even when the coupling was very important, it was found, for SWATH's 4A and 4B, that the period of one of the modes was always close in value to the uncoupled heave period and was easily identifiable. It was, therefore, meaningful in this report to refer to "heave" and "pitch" modes. The uncoupled periods are readily shown to be

for heave $$T_{h0} = 2\pi \left[\frac{C_{33}}{M + A_{33}} - \frac{B_{33}^2}{4(M + A_{33})^2} \right]^{-1/2} \tag{11.1}$$

for pitch $$T_{p0} = 2\pi \left[\frac{C_{55}}{I_5 + A_{55}} - \frac{B_{55}^2}{4(I_5 + A_{55})^2} \right]^{-1/2} \tag{11.2}$$

These have meaning only when the quantities under the radicals are positive, i.e., the mode is oscillatory. This was always true for the heave equation and almost always true for the pitch part.

By analogy with the simple oscillator, each mode can be thought of as having a damping ratio ζ_n which is defined in terms of the pair of roots describing that mode. Thus, if λ_1 and λ_2 (Equations (8)) are identified as the roots of the heave mode then

$$\zeta_3 = -\frac{\lambda_1 + \lambda_2}{2\sqrt{\lambda_1 \lambda_2}} \tag{12.1}$$

and similarly for the pitch mode

$$\zeta_5 = -\frac{\lambda_3 + \lambda_4}{2\sqrt{\lambda_3 \lambda_4}} \tag{12.2}$$

Another measure of the degree of stability is the time that must elapse for the disturbance motion of each mode to double or halve itself. In the case of oscillatory modes it is the envelope of the disturbance that doubles or halves. It is easy to show that this time is given by

$$T^{(1/2)} \text{ or } T^{(2)} = \ell n \; 2/|\lambda_{nR}| \tag{12.3}$$

Clearly as a stable mode becomes progressively less stable, the value of $T^{(1/2)}$ becomes larger and larger. It is generally the root with the smallest real part that is critical.

The corresponding values of the damping ratio and the time to half amplitude for the uncoupled heave and pitch modes, respectively, can, of course, be written much more simply in terms of the coefficients of the equations of motion. They are given as follows for ready reference.

$$\zeta_{h0} = \frac{B_{33}}{2} [(M + A_{33}) C_{33}]^{-1/2} \tag{13.1}$$

$$\zeta_{p0} = \frac{B_{55}}{2} [(I_5 + A_{55}) C_{55}]^{-1/2} \tag{13.2}$$

$$
T_{h0}^{(1/2)} = \begin{cases} 2(M + A_{33})\,\ell n\, 2/B_{33} & \text{for } \zeta_{h0} < 1 \\[2ex] 2(M + A_{33})\,\ell n\, 2 \Bigg/ \left[B_{33} - \left\{ B_{33}^2 - 4(M + A_{33})\, C_{33} \right\}^{1/2} \right] & \text{for } \zeta_{h0} > 1 \end{cases} \tag{13.3}
$$

$$
T_{p0}^{(1/2)} = \begin{cases} 2(I_5 + A_{55})\,\ell n\, 2/B_{55} & \text{for } \zeta_{p0} < 1 \\[2ex] 2(I_5 + A_{55})\,\ell n\, 2 \Bigg/ \left[B_{55} - \left\{ B_{55}^2 - 4(I_5 + A_{55})\, C_{55} \right\}^{1/2} \right] & \text{for } \zeta_{p0} > 1 \end{cases} \tag{13.4}
$$

In addition to the transient response, the frequency response of the craft to sinusoidal waves, which is often represented by response amplitude operator (RAO), is, of course, also very much affected by the stability roots. Much insight can be obtained from an examination of the equations for RAO expressed in terms of the stability roots, especially at the important encounter frequencies near resonance.

If we replace the right-hand sides of Equations (1) and (2) by the wave-exciting terms $F_3 e^{i\omega t}$ and $F_5 e^{i\omega t}$, respectively, then we can show that the heave and pitch motion are, respectively, given by

$$
\xi_3 = \frac{F_3' \, e^{i\omega t}}{a\,(i\omega - \lambda_1)(i\omega - \lambda_2)(i\omega - \lambda_3)(i\omega - \lambda_4)} \tag{14.1}
$$

$$
\xi_5 = \frac{F_5' \, e^{i\omega t}}{a\,(i\omega - \lambda_1)(i\omega - \lambda_2)(i\omega - \lambda_3)(i\omega - \lambda_4)} \tag{14.2}
$$

where a is defined by Equation (7.1), and

$$
F_3' = F_3 \left\{ -\omega^2 (I_5 + A_{55}) + i\omega B_{55} + C_{55} \right\} - F_5 (-\omega^2 A_{35} + i\omega B_{35} + C_{35})
$$

$$
F_5' = F_5 \left\{ -\omega^2 (M + A_{33}) + i\omega B_{33} + C_{33} \right\} - F_3 (-\omega^2 A_{53} + i\omega B_{53} + C_{53})
$$

and λ_i, $i = 1,\ldots 4$ are the stability roots.

As will be seen later, the heave mode roots λ_1, λ_2 for SWATH's 4A and 4B are complex conjugates for all the conditions of interest. Furthermore, this mode is significantly less stable than the pitch mode. We, therefore, take a closer look at Equations (14.1) and (14.2)

at wave-encounter frequencies near the damped natural frequency of the heave mode. We rewrite these equations as follows

$$\xi_3 = \frac{F_3'}{aD} e^{i(\omega t - \epsilon_1(\omega) - \epsilon_2(\omega))} \tag{15.1}$$

$$\xi_5 = \frac{F_5'}{aD} e^{i(\omega t - \epsilon_1(\omega) - \epsilon_2(\omega))} \tag{15.2}$$

where $D = \omega_{0H}^2 \lambda_3 \lambda_4 \left[\left(1 - \frac{\omega^2}{\omega_{0H}^2}\right)^2 + 4\frac{\omega^2}{\omega_{0H}^2}\zeta_3^2\right]^{1/2} \left[\left(1 - \frac{\omega^2}{\lambda_3\lambda_4}\right)^2 + 4\frac{\omega^2}{\lambda_3\lambda_4}\zeta_5^2\right]^{1/2}$

$\omega_{0H}^2 = \lambda_1 \lambda_2$ = square of undamped natural frequency of heave mode

$\zeta_3^2 = \frac{(\lambda_1 + \lambda_2)^2}{4\lambda_1\lambda_2}$ $\left(\text{or } \zeta_5^2 = \frac{(\lambda_3 + \lambda_4)^2}{4\lambda_3\lambda_4}\right)$ = square of damping ratio of heave (or pitch) mode (Equations (12.1) and (12.2))

$\epsilon_1(\omega)$ and $\epsilon_2(\omega)$ = arguments of heave- and pitch-mode terms, respectively, in the denominator

It can be seen from the previous equations that the term in the first bracket in denominator D can have a sharp minimum, depending on the magnitude of the damping ratio ζ_3. If the second bracketed term in D varies relatively slowly with frequency, then heave and pitch RAO might be expected to exhibit a peak at the heave resonant frequency with characteristics mainly determined by the heave roots. As may be inferred from Equation (15), sharpness of the peak would decrease with increasing ζ_3. The frequency at which this peak occurs is given by

$$\omega_H = \omega_{0H}\sqrt{1 - 2\zeta_3^2} = \lambda_{II}\left[\frac{1 - 2\zeta_3^2}{1 - \zeta_3^2}\right]^{1/2} \quad \text{for } \zeta_3 < \frac{1}{\sqrt{2}}$$

where λ_{II} is the damped natural frequency of the heave mode, and ω_H is essentially the resonant frequency for the heave mode. It is clear from the previous equation that the

resonance frequency is smaller than both the undamped and damped natural frequencies. Substituting this expression for ω_H in the bracketed terms of Equations (15.1) and (15.2) leads to the following equations for the approximate peak response.

$$\xi_3 = \frac{F'_3}{2aD_H} e^{i(\omega t - \epsilon_1(\omega) - \epsilon_2(\omega))} \tag{15.3}$$

$$\xi_5 = \frac{F'_5}{2aD_H} e^{i(\omega t - \epsilon_1(\omega) - \epsilon_2(\omega))} \tag{15.4}$$

where

$$D_H = \omega_{0H}^2 \lambda_3 \lambda_4 \zeta_3 \sqrt{1 - \zeta_3^2} \left[\left\{ 1 - \frac{\omega_{0H}^2}{\lambda_3 \lambda_4} (1 - 2\zeta_3^2) \right\}^2 + 4 \frac{\omega_{0H}^2}{\lambda_3 \lambda_4} (1 - 2\zeta_3^2) \zeta_5^2 \right]^{1/2} \tag{15.5}$$

and

$$\zeta_3 < \frac{1}{\sqrt{2}}$$

The quantity $\dfrac{1}{2\zeta_3 \sqrt{1 - \zeta_3^2}}$ is also an approximate indication of the magnification factor of the response relative to the static response ($\omega = 0$) which, according to Equations (15), would be approximately proportional to $\dfrac{1}{a\omega_{0H}^2 \lambda_3 \lambda_4}$.

As has been noted, the foregoing statements are reasonably correct, provided the numerator and the terms containing λ_3, λ_4 in the denominator are not overly sensitive to frequency in the vicinity of heave resonance. This condition is satisfied in part by the vehicles considered here, especially for the conditions in which the pitch mode is significantly more stable than the heave mode. It should also be noted from Equations (14) that the pitch motion will not have as sharp a peak as the heave motion since the pitch-excitation term tends to approach a minimum in the vicinity of the heave resonance when the heave damping is low; see equation for F'_5 in Equation (14.2). Similar considerations to those given previously for the heave mode would also apply to the pitch mode. Thus, if both the pitch and heave modes have low damping ratios, then the frequency response will show, in general, two

resonant peaks, corresponding to resonant encounter frequencies in heave and pitch modes. This is true, provided the resonant frequencies are reasonably well separated. When they are close together, and the damping is low, a much smaller minimum in the denominator of Equations (15) can occur. Although one may, therefore, expect a sharper resonance peak, this can be modified appreciably by the fact that F_3' and F_5' also may get smaller near resonance.

One motion criterion of great importance is represented by the magnitude of the relative bow motion with respect to the wave surface. This can be obtained by

$$R = |\xi_3 \quad \ell_a \xi_5 \quad \eta|$$

where $\eta = A\, e^{i(\omega^2/g\ \ell_a + \omega t)}$ is the wave elevation at the bow, and ℓ_a is the x coordinate of the bow.

We can express the relative bow motion in a nondimensional form as

$$\frac{R}{A} = \left| \frac{\xi_3}{A} \quad \frac{\ell_a \xi_5}{A} \quad e^{i(\omega^2/g\ \ell_a + \omega t)} \right| \tag{16}$$

The value of R/A depends on the amplitudes and the phases of heave, pitch, and wave motions. However, if we choose fins which provide good stability and small heave and pitch amplitudes, we are more likely to have smaller relative bow motions. From Equations (15.3) and (15.4) we have seen that the amplitudes of heave and pitch motions $|\xi_3|$ and $|\xi_5|$ at the resonance of the heave mode increase rapidly as the damping ratio decreases. The same is, of course, true for the pitch mode. By increasing the size of the stabilizing aft fins, it will be seen that adequate pitch-mode damping can be readily obtained while the same is not necessarily true for the heave-mode damping. Thus, our aim should be directed toward obtaining a fin configuration which would also provide suitable heave damping and a natural frequency so that the heave amplitude at the heave resonance becomes minimum.

NUMERICAL RESULTS

Input data necessary to obtain the stability roots λ_n from Equation (6) are given in Table 2. The subscripted coefficients B* are obtained from available experimental data for submarine models, and the remainder of the subscripted coefficients A and B are obtained from potential flow theory,[5] including free surface.

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[5] Pien, P. C. and C. M. Lee, "Motion and Resistance of a Low-Waterplane-Area Catamaran," The Ninth Naval Hydrodynamic Symposium, Paris (1972).

TABLE 2 INPUT DATA

	Nondimensionalization Factor	SWATH 4A	SWATH 4B
ρ	1	2.0	2.0
g	1	32.17	32.17
$\forall$ (Displaced Volume)	1	83,027 ft^3	83,027 ft^3
$L_{\forall}$	1	208.3 ft	247.3 ft
L (Overall Length)	1	239.7 ft	275.9 ft
A_{wp}	$\forall / L_1$	4.732	5.746
M_{wp}	$\forall$	−0.0495	−0.0475
$\overline{(GM)}_{\ell}$	L_1	0.133	0.236
I_5	$\rho \forall L_1^2$	0.0625	0.0625
A_{33}	$\rho \forall$	0.629	0.668
A_{35} (= A_{53})	$\rho \forall L_1$	0.00517	0.00409
A_{55}	$\rho \forall L_1^2$	0.0585	0.0587
$B_{33}^{(0)} \int_L b_{33}(x)dx$	$\rho \forall \sqrt{gL_1}$	0.111	0.0885
$B_{35}^{(0)} = \int_L (-x)\, b_{33}(x)dx$	$\rho \forall \sqrt{gL_1}$	0.000582	0.000369
$B_{55}^{(0)} = \int_L x^2 b_{33}(x)dx$	$\rho \forall L_1 \sqrt{gL_1}$	0.00567	0.00486
$B_{33}^{\bullet}$	$\frac{\rho}{2} L^2$	0.00494	0.0036
$B_{35}^{\bullet}$	$\frac{\rho}{2} L^3$	0.0010	0.00078
$B_{55}^{\bullet}$	$\frac{\rho}{2} L^4$	0.0004	0.00026
$B_{53}^{\bullet}$	$\frac{\rho}{2} L^3$	−0.006	−0.004
$C_{L\alpha}$	1	3.0	3.0
ℓ_1	1	−85.0 ft	−101.[illegible] ft
ℓ_2	1	80.25 ft	101.9 ft

One of the conditions for stability, according to Routh in Equation (9), is that

$$e > 0$$

On substituting Equation (7.5) for e, we obtain

$$C_{55} > C_{35}C_{53}/C_{33} \tag{17.1}$$

From Equations (3) and (17.1) we have a speed condition for stability for the bare hull case as follows

$$U_0 < \left[\frac{\Delta \overline{GM}_{\ell}}{\frac{\rho}{2} L^3 M'_w} \quad \frac{C_{35}C_{53}}{\frac{\rho}{2} L^3 M'_w C_{33}} \right]^{1/2} \tag{17.2}$$

which states that a sufficient condition for instability is that the speed be greater than the right hand side. Since $\Delta \overline{GM}_{\ell} \gg \dfrac{C_{35}C_{53}}{C_{33}}$ for SWATH's 4A and 4B, as can be shown by substituting the values from Table 2 for the coefficients, Equation (17.2) may be simplified to

$$U_0 < \left[\frac{\Delta \overline{GM}_{\ell}}{\frac{\rho}{2} L^3 M'_w} \right]^{1/2} \tag{17.3}$$

This amounts to approximating the inequality (17.1) by

$$C_{55} > 0 \tag{17.4}$$

The computed values of U_0 for SWATH's 4A and 4B are, respectively, 25.1 and 36.0 knots. The larger value of U_0 for SWATH 4B is mainly attributable to the larger $\overline{GM}_{\ell}$, which is 58.4 feet, compared to 27.7 feet for SWATH 4A. Values of U_0 found in this manner are very close to those obtained from the coupled equations.

To maintain a reasonable stability to 40 knots, both ships would need a stabilizing fin or fins. To determine the appropriate fin size, a first approximation for the lower bound is made from Equation (3.12) to satisfy $C_{55} > 0$ at 40 knots. That means

$$\Delta\,\overline{GM}_{\ell} - U^2 \frac{\rho}{2} L^3 M'_w - \sum_{i=1}^{N} \frac{\rho}{2} U^2 c_i s_i \ell_i\, C_{L\alpha i} > 0$$

For N = 2, i.e., one fin for each hull of identical size, and $\ell_1 = \ell_2 < 0$, we have for the lower bound

$$c_1 s_1\, C_{L\alpha 1} = \frac{\frac{\rho}{2} U^2 L^3 M'_w - \Delta\,\overline{GM}_{\ell}}{\rho\, U^2\, |\ell_1|} \tag{18}$$

for U = 40 knots. Although the fin size obtained in this manner would be inadequate, it can be used to represent the lower bound of the range of fin sizes to be investigated.

With the aid of earlier estimates for the SWATH 4 fins, obtained in this manner, estimates for SWATH 4A and SWATH 4B showed that the following fins are reasonably good sizes to insure pitch stability

	Chord ft	Span ft	Location
SWATH 4A	11.9	14.3	Inboard side of each hull at 0.84 L
SWATH 4B	10.0	12.0	Inboard side of each hull at 0.85 L

It should be stressed that what is really being optimized in this investigation is the product $c_i s_i\, C_{L\alpha i}$. Clearly this quantity may be realized by an infinite number of fin plan forms. The plan form described previously is felt to be conservative on the basis of strength considerations. Optimization of the plan form for minimum drag for various values of $c_i s_i\, C_{L\alpha i}$ is beyond the scope of this investigation; it should be done in the preliminary design stages. These fins are designated 1.0 for SWATH 4A and SWATH 4B, respectively. Other fins with the same aspect ratio but different sizes are designated with numbers indicating the projected fin-area ratio to the 1.0 fin. For instance, the 0.0 fin means a bare hull.

For each fin, the four roots of Equation (6) are found for various speeds, and the natural periods, the half-decay time and the damping ratios are computed from Equations (10) and (12). The values are shown in Table 3 for SWATH 4A and in Table 4 for SWATH 4B for various fin sizes at the speeds of 30, 35, and 40 knots.

TABLE 3 TRANSIENT CHARACTERISTICS OF SWATH 4A

Fin Size		Mode	Damping Ratio			Natural Period sec			Half Decay Time sec		
Aft	Fore		30 knots	35 knots	40 knots	30 knots	35 knots	40 knots	30 knots	35 knots	40 knots
0	0	Heave	0.02*	0.02**		9.36*	8.82**		51.93*	13.20**	
		Pitch	0.02*	0.23**		15.31*	28.05**		74.95*	50.43**	
0.85	0	Heave	0.18	0.20	0.22	9.15	9.09	9.02	5.58	4.88	4.34
		Pitch	0.65	0.80	0.95	23.51	31.80	125.23	3.02	2.60	2.28
1.0	0	Heave	0.22	0.25	0.29	9.66	9.78	9.92	4.76	4.12	2.28
		Pitch	0.61	0.70	0.79	19.05	21.08	24.41	2.72	2.35	2.07
1.05	0	Heave	0.23	0.27	0.31	9.91	10.14	10.44	4.57	3.98	3.53
		Pitch	0.60	0.68	0.75	17.81	18.84	20.18	2.62	2.26	1.98
1.10	0	Heave	0.25	0.29	0.32	10.21	10.61	11.15	4.46	3.93	3.59
		Pitch	0.59	0.66	0.71	16.63	16.89	17.06	2.51	2.14	1.85
1.15	0	Heave	0.25	0.29	0.31	10.58	11.19	11.90	4.44	4.11	4.04
		Pitch	0.58	0.65	0.71	15.52	15.24	14.98	2.38	1.98	1.66
1.20	0	Heave	0.25	0.27	0.27	11.01	11.70	12.35	4.62	4.61	4.83
		Pitch	0.59	0.65	0.72	14.51	14.11	13.09	2.22	1.80	1.49
1.50	0	Heave	0.14	0.12	0.09	12.14	12.58	13.00	9.21	11.40	15.14
		Pitch	0.69	0.75	0.79	12.47	12.22	11.93	1.44	1.19	1.01
2.0	0	Heave	0.03	0.01	Unstable	12.55	12.94	Unstable	49.26	112.88	Unstable
		Pitch	0.81	0.85		12.24	11.94		0.99	0.83	
1.8	0.2	Heave	0.20	0.17	0.14	12.83	13.51	13.48	7.05	8.50	11.04
		Pitch	0.80	0.85	0.88	13.81	13.71	14.13	1.14	0.94	0.80
1.6	0.4	Heave	0.38	0.44	0.50	11.46	12.59	14.21	3.05	2.80	2.74
		Pitch	0.89	0.96	1.0	26.05	37.68	173.24	1.46	1.22	1.03
1.3	0.7	Heave	0.31	0.34	Unstable	9.0	8.91	Unstable	3.09	2.70	Unstable
		Pitch	0.54	0.51					9.80	23.87	

*Zero Speed
**20 knots

TABLE 4 TRANSIENT CHARACTERISTICS OF SWATH 4B

Fin Size		Mode	Damping Ratio			Natural Period sec			Half Decay Time sec		
Aft	Fore		30 knots	35 knots	40 knots	30 knots	35 knots	40 knots	30 knots	35 knots	40 knots
0	0	Heave	0.01	0.01	--	8.49	8.34	–	66.47	80.73	--
		Pitch	0.25	0.78		26.13	103.68		10.98	9.37	–
0.36	0	Heave	0.05	0.06	0.06	8.51	8.34	8.18	17.17	16.26	15.76
		Pitch	0.35	0.52	0.96	20.53	28.28	147.99	6.12	5.19	4.47
0.67	0	Heave	0.10	0.11	0.11	8.68	8.53	8.37	9.74	8.75	8.01
		Pitch	0.39	0.51	0.67	17.66	20.95	28.06	4.58	3.91	3.39
1.0	0	Heave	0.15	0.17	0.19	9.11	9.05	8.98	6.58	5.73	5.09
		Pitch	0.42	0.50	0.59	15.15	16.44	18.36	3.64	3.13	2.75
1.1	0	Heave	0.17	0.19	0.22	9.34	9.34	9.33	6.04	5.24	4.62
		Pitch	0.42	0.50	0.57	14.38	15.23	16.40	3.40	2.93	2.58
1.2	0	Heave	0.18	0.21	0.24	9.66	9.77	9.90	5.68	4.94	4.38
		Pitch	0.43	0.49	0.56	13.54	13.96	14.48	3.15	2.70	2.37
1.32	0	Heave	0.19	0.22	0.24	10.12	10.39	10.69	5.70	5.17	4.81
		Pitch	0.44	0.51	0.57	12.65	12.70	12.79	2.82	2.38	2.04
1.5	0	Heave	0.16	0.17	0.17	10.73	11.03	11.34	7.19	7.10	7.10
		Pitch	0.50	0.57	0.64	11.73	11.71	11.71	2.22	1.84	1.57
2.0	0	Heave	0.08	0.07	0.06	11.22	11.53	11.84	15.03	17.73	22.84
		Pitch	0.65	0.71	0.76	11.25	11.18	11.08	1.47	1.23	1.06
1.8	0.2	Heave	0.2	0.22	0.23	10.99	11.43	11.90	5.86	5.68	5.65
		Pitch	0.62	0.69	0.75	12.43	12.62	12.84	1.74	1.45	1.23
1.6	0.4	Heave	0.26	0.30	0.34	9.79	9.94	10.12	3.99	3.45	3.04
		Pitch	0.68	0.78	0.89	16.81	19.89	27.08	2.02	1.74	1.52
1.3	0.7	Heave	0.22	0.24	0.27	8.75	8.61	8.46	4.30	3.77	3.38
		Pitch	0.97	0.60	0.54	68.96	∞	∞	1.96	4.64	8.87

Figure 2 shows the locus of the stability roots* obtained from Equation (6) for SWATH 4A at 30 knots. The blank circles with the numbers indicating the fin size are for a pair of aft fins. The solid circles are the stability roots for two pairs of fins; for each hull, one pair is located near the bow, and the other is located near the stern. Figures 3 and 4, respectively, are similar plots to Figure 2 for 35 and 40 knots. As noted earlier, the vehicle is stable as long as the real part of the root is negative.

The bases of selecting the most desirable fin size are, as stated earlier, to insure stability, to obtain adequate damping for heave and pitch, and to maintain reasonable natural periods. As can be observed in Figures 2 through 4, except for the bare hull case, the aft fins ranging from 0.7 to 2.0 appear to provide necessary stability in the pitch mode. The larger the fin, the larger the absolute value of the real root for pitch. This trend is not true for the heave mode. At approximately the 1.1 fin, $|\lambda_R|$ takes a maximum value and decreases as the fin size increases or decreases. From Equation (12), it can easily be shown that, when the roots are complex, the damping ratio is obtained by the cosine of the angle between the negative real axis, and the line connecting the root to the origin; vis., $\zeta = \cos\gamma$ in Figure 2 for the damping ratio of the heave mode using the 1.15 fin; see also Figure 10. We find that in the case of aft fin only, the 1.1 fin provides the maximum damping ratio for SWATH 4A between 30 and 40 knots. In this speed range, the natural period for heave is about 10.5 seconds and for pitch about 16.8 seconds. These periods correspond to encountering head sinusoidal waves of 1400 and 2900 feet, respectively, at 30 knots. This indicates that the natural periods are reasonably large, and, furthermore, they are well separated. The natural periods for heave and pitch at zero speed are 9.4 and 15.3 seconds, respectively.

The case of using stabilizing fins both fore and aft was investigated to a limited extent. As can be noted from Figures 2 through 4 for the case of aft fins alone, increase of aft fin size after a certain size causes a decrease in heave stability as well as heave damping ratio. This fact implies that a possible way of obtaining a large heave damping ratio for a given projected fin area is to divide the fin area into fore and aft fins. However, since the fore fins generate a destabilizing pitch moment, the aft fins should be sufficiently large to compensate the destabilizing moment induced by the fore fins.

The total fin area was kept twice that of the 1.0 fin but was divided into fore and aft fins of the same aspect ratio. The forward fin was assumed to be located at approximately the 0.15 L position at the inboard side of each hull and the aft fin at the 0.84 L position at the inboard side of each hull. The numbers beside the solid circles indicate the aft fin size. Hence, the size of the fore fin is two minus the number indicating the aft fin size. As can be

*Only the positive imaginary part of the complex roots are shown.

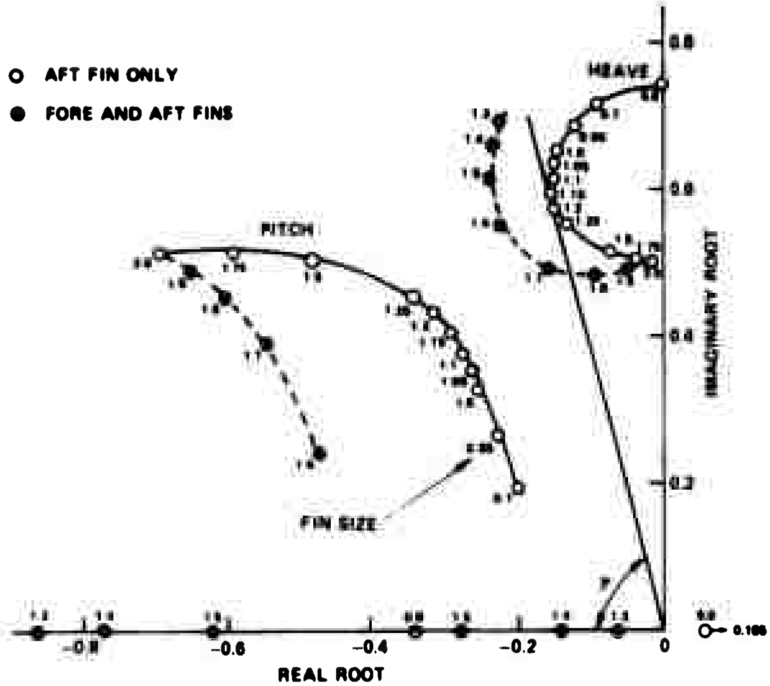

Figure 2 – Stability Roots for Various Fin Sizes for SWATH 4A at 30 Knots

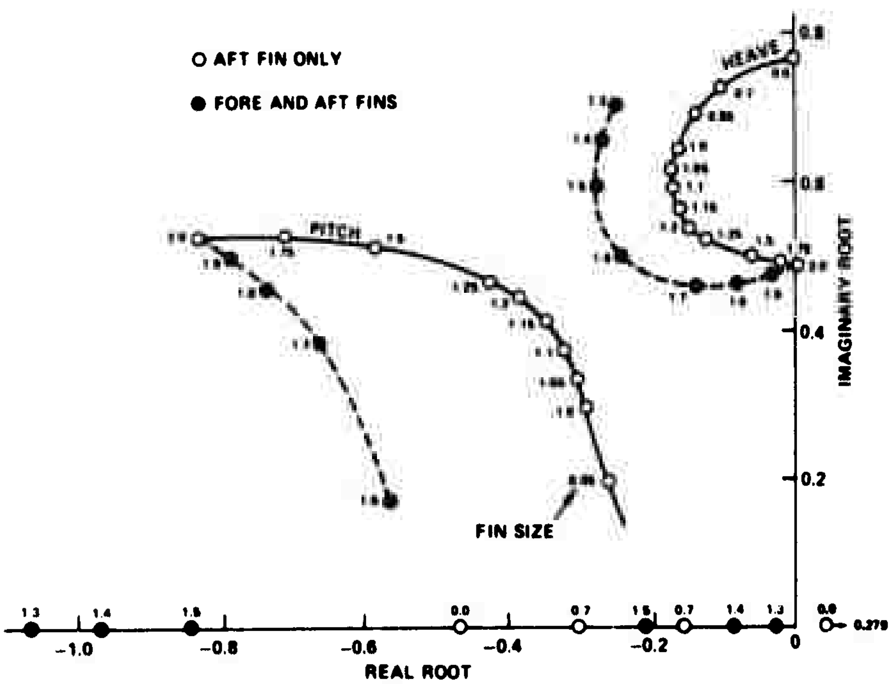

Figure 3 – Stability Roots for Various Fin Sizes for SWATH 4A at 35 Knots

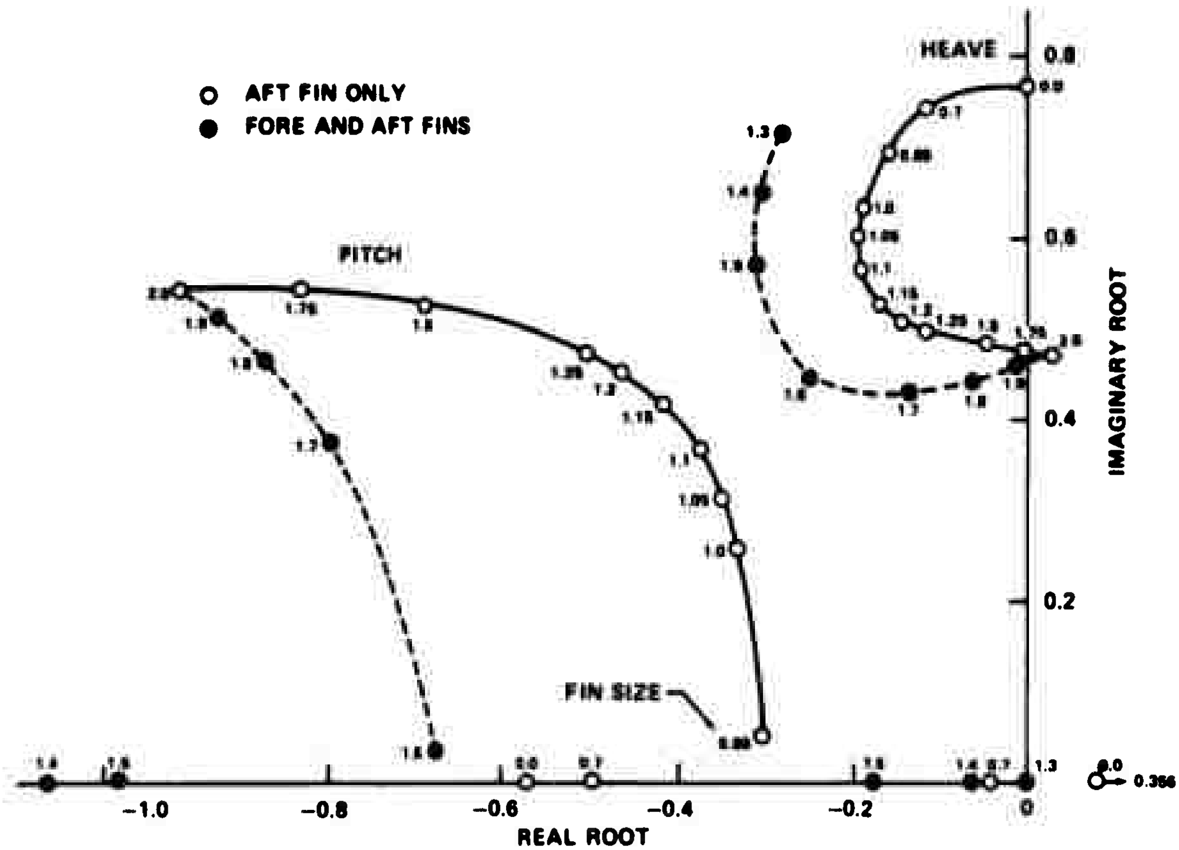

Figure 4 – Stability Roots for Various Fin Sizes for SWATH 4A at 40 Knots

seen from these figures, the heave damping is markedly increased by the use of the fore-and-aft fin combination. However, the pitch stability begins to decrease with the aft fins less than the 1.6 fin as speed increases.

Since the use of fore and aft fins may lead to significantly higher drag, feasibility should be determined in terms of the advantages they may offer as far as improvement in ship motions and control is concerned. This is beyond the scope of the present investigation.

The stability roots for SWATH 4B at 30, 35, and 40 knots are shown in Figures 5 through 7. For the aft fin only cases, the size of the aft fins investigated range from 0.36 to 2.0 fin. For the fore and aft fin cases, the sizes of the fore and aft fins are chosen like SWATH 4A by keeping the total projected fin area to twice that of the 1.0 fin of SWATH 4B. The numbers beside the solid circles indicate the size of the aft fin of the fore-and-aft fin combination. The aft fins are located at the 0.85 L position, and the fore fins are located at the 0.12 L position.

SWATH 4B without stabilizing fins has a speed of inception for the pitch mode instability of 36 knots. As can be seen from Figures 5 through 7, the 0.0 fin shows negative real roots for pitch at 35 knots, implying that the boat is stable in pitch even at this high speed. This is due to the large longitudinal metacentric height $\overline{GM}_{\ell}$ that contributes directly to restoring moment of pitch. However, an increase in $\overline{GM}_{\ell}$ decreases the natural period of pitch at zero speed. As the speed increases, the destabilizing Munk moment increases rapidly as the square of the speed, and, hence, the effective restoring moment for pitch decreases. This results in an increase of the natural period. With a stabilizing fin, the loss of restoring moment due to the Munk moment is compensated to some extent, and the natural period does not change with speed too much. In fact it is possible with a large enough fin to cause a reduction in the natural period of pitch with speed; see Table 4.

In the case of aft fin alone, the fin which provides a maximum heave mode damping is found from Figures 5 through 7 to be the 1.2 fin for SWATH 4B. The natural periods for heave and pitch in the 30- to 40-knot range are, respectively, 9.8 and 14 seconds. Although these are reasonably large natural periods, the gap between the two periods is not as large as that for SWATH 4A. The natural periods at zero speed are 9.4 seconds for heave and 12.5 seconds for pitch. Thus, with the 1.2 fin, SWATH 4B can widen the gap between the natural periods of heave and pitch as speed increases. If we place more emphasis on the gap between the two natural periods than on the heave damping, probably the 1.0 fin would be a better choice. As can be seen from Table 4 for the 1.2 and 1.0 fins, the damping ratio for heave at 30 knots reduces from 0.18 for the 1.2 fin to 0.15 for the 1.0 fin. The corresponding time to half-amplitude has also gone up approximately 20 percent, while the difference between the two natural periods increases from 4.08 to 6.04 seconds.

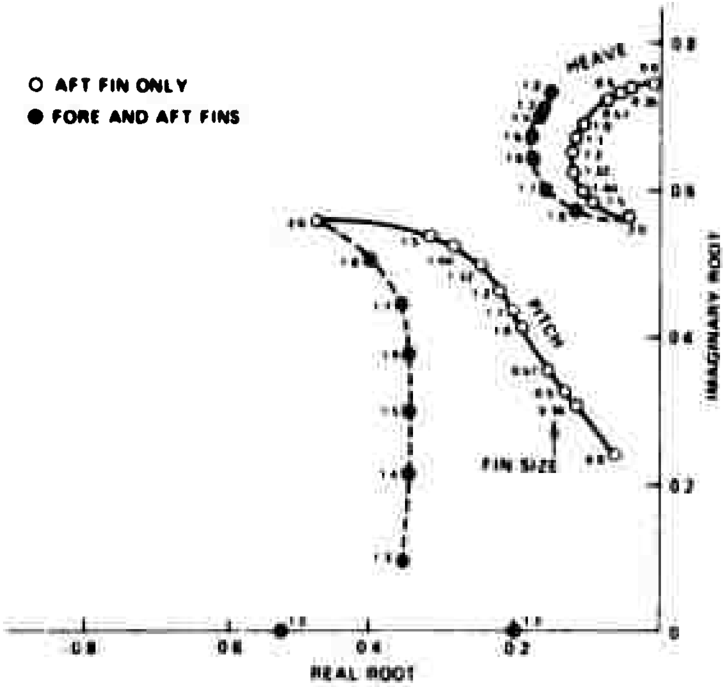

Figure 5 Stability Roots for Various Fin Sizes for SWATH 4B at 30 Knots

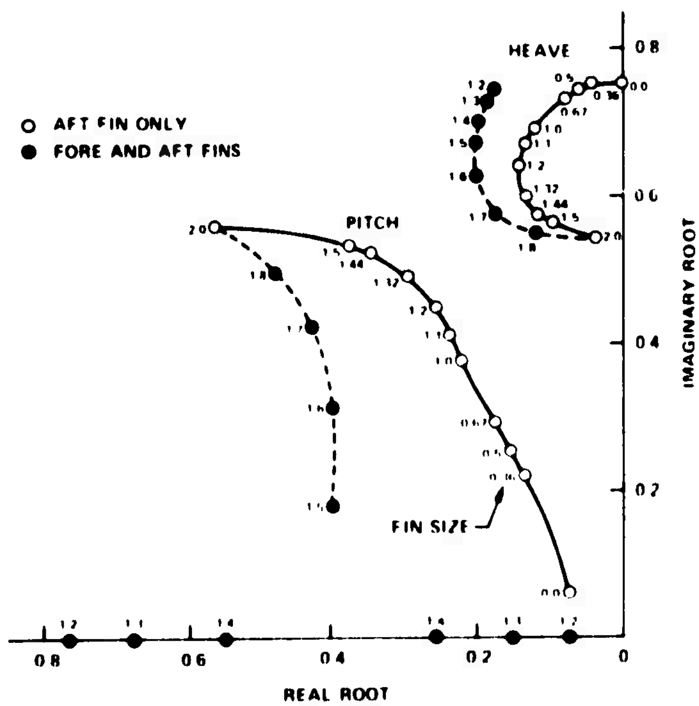

Figure 6 Stability Roots for Various Fin Sizes for SWATH 4B at 35 Knots

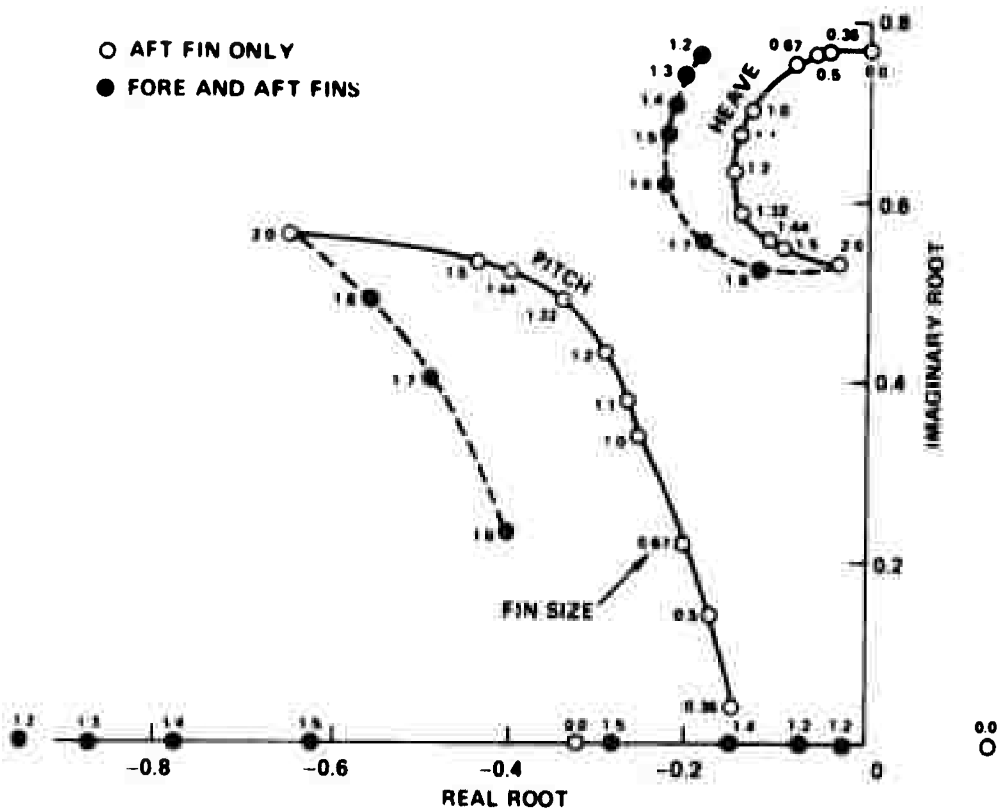

Figure 7   Stability Roots for Various Fin Sizes for SWATH 4B at 40 Knots

The fore and aft fins for SWATH 4B also demonstrate an increase in pitch stability and heave damping. The increase appears not so pronounced as in the case of SWATH 4A. The combination of the 0.4 fin forward and 1.6 fin aft shows a maximum heave mode damping ratio.

In the range from 30 to 40 knots, the stabilizing fins have more effect on SWATH 4A than on SWATH 4B. This phenomenon may be expected, since in this speed range, SWATH 4A can maintain its stability only by using stabilizing fins, whereas SWATH 4B is stable to 36 knots without fins as a result of the larger stiffness contributed by the lengthened struts. In other words, the stiffer the ship the harder it is to influence or control its initial characteristics. If the fins are controllable, the effectiveness of the fins may be expected to be significantly greater for SWATH 4A than SWATH 4B.

The effect of speed on the transient characteristics, damping ratio, natural period, and half-decay time for SWATH 4A with the 1.1 fin and SWATH 4A with 1.2 fin are shown in Figures 8 and 9. It is seen that by installing the stabilizing fins, not only is the craft stabilized but stability is also improved with increasing speed. It appears that SWATH 4A has somewhat better overall stability characteristics. Although this will probably result in better response to the seaways of interest, it would be necessary to determine this separately.

To study the effect of fins on the motion characteristics of ships in waves, the heave and pitch motions in regular head waves were examined. Since our major interest lay in determining the critical motion behavior, the study of the motion was limited to the heave resonant frequency region. A ship speed of 35 knots was chosen for this study.

If we exar ine the heave and pitch motion near the heave resonance given by Equations (15), we can infer that to have minimum values of $|\xi_3|$ and $|\xi_5|$, we should have minimum values of $|F'_3|$ and $|F'_5|$ as well as a maximum value of D_H. Since the effect of fins on the wave-exciting terms F'_3 and F'_5 is very small, the minimization of $|\xi_3|$ and $|\xi_5|$ should be accomplished by maximizing the denominator D_H on which the fins have a direct influence.

We can write D_H given by Equation (15.5) as

$$D_H = 2\,\omega_{0H}^2\,\zeta_3\sqrt{1-\zeta_3^2}\,|(i\omega_H-\lambda_3)(i\omega_H-\lambda_4)|$$

where

$$\omega_H = \omega_{0H}\sqrt{1-2\zeta_3^2} \text{ for } \zeta_3 < \frac{1}{\sqrt{2}}$$

is the frequency at which the heave motion would peak, if the magnitude of the term in brackets were to vary slowly with frequency near this frequency. The undamped heave

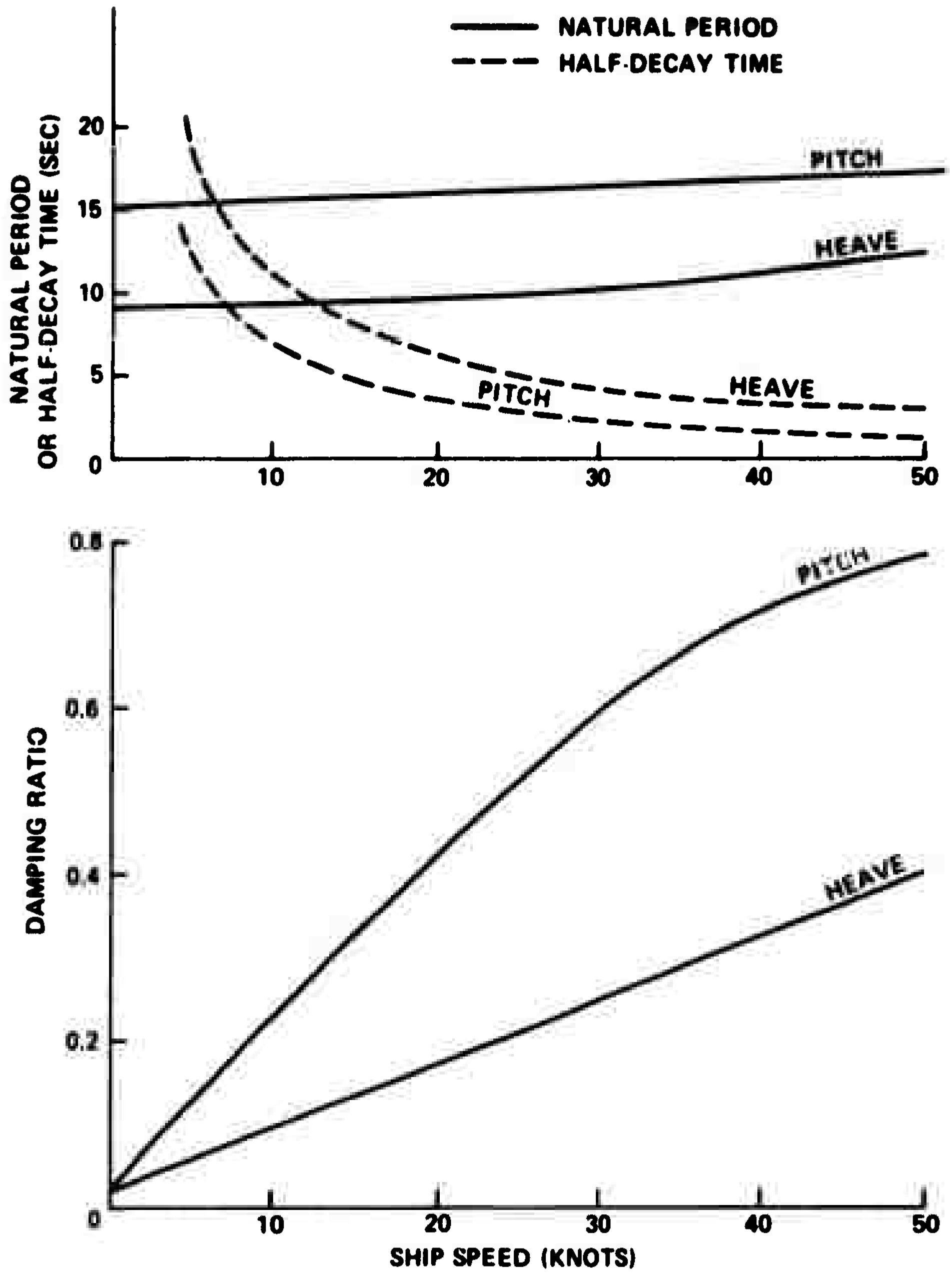

Figure 8 - Transient Characteristics versus Speed for SWATH 4A with 1.1 Fin

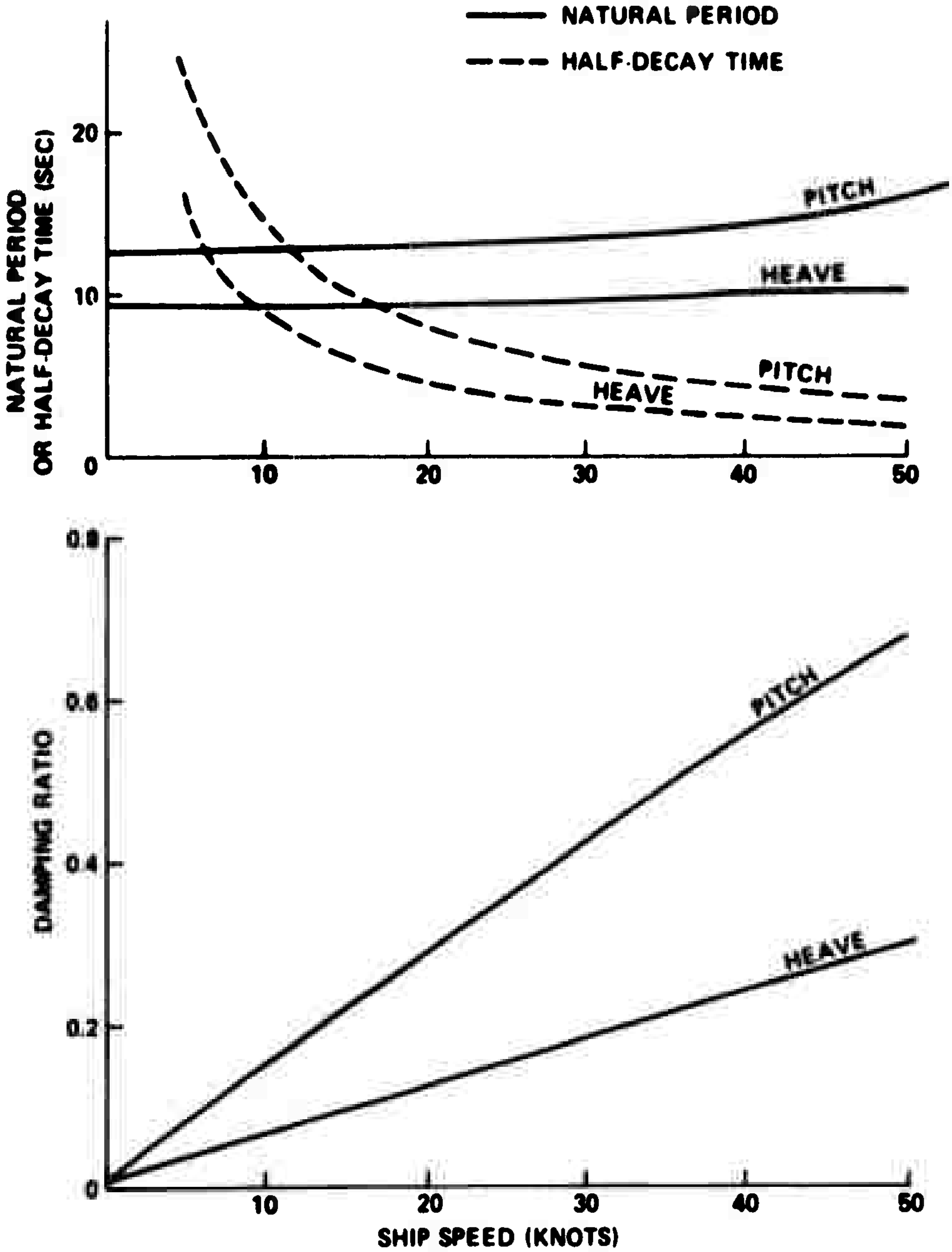

Figure 9 - Transient Characteristics versus Speed for SWATH 4B with 1.2 Fin

natural frequency ω_{0H} and the heave damping ratio ζ_3 are functions of the stability roots λ_1 and λ_2 as shown earlier. The maximization of D_H may be pursued in terms of the stability roots of the heave mode alone, if it yields a correct answer. For this purpose, we define a new denominator, which ignores the effect of the pitch mode.

$$D_{H0} = \omega_{0H}^2 \zeta_3 \sqrt{1 - \zeta_3^2} = |\lambda_{1R} \lambda_{1I}| \tag{19}$$

In Table 5 the values of D_H and D_{H0} are shown for different configurations of fin for SWATH 4A and SWATH 4B. Fin configurations which are judged to provide insufficient pitch mode stability are not considered. As can be observed in Table 5, the fins which provide a maximum value of D_{H0} do not necessarily provide a maximum value of D_H. This implies that the effect of pitch mode roots on the peak value of heave amplitude is not negligible. As far as the value of D_{H0} is concerned, the fins that provide a maximum heave damping ratio yield a maximum value of D_{H0}. From Equation (15.5) we note that $\lambda_3 \lambda_4$ is also an important factor to be considered in the maximization of D_H. For those fin configurations considered in Table 5, λ_3 and λ_4 are a pair of complex conjugate roots. Thus, $\lambda_3 \lambda_4 = |\lambda_3|^2$, and we can denote this by ω_{0P}^2, which is the square of the undamped natural frequency of the pitch mode. To best visualize the relation between the stability roots and the various quantities introduced in the present analysis, a vector diagram is given in Figure 10. As can be seen, the product of the magnitudes of the vectors from the point $i\omega$ to each of the four roots is simply D_H; see Equations (14). Figure 10 thus shows the effect of the root distribution on the magnitude of D_H.

It would be still premature to expect that the peak value of the heave amplitude could be a minimum, when the value of D_H is a maximum. We have to include the effect of the wave-exciting terms to judge which fin configuration yields the most desirable motion in waves.

Figure 11 shows the heave amplitude and the relative motion amplitude of the hull at the 0.06 L position for SWATH 4A and at the 0.05 L position for SWATH 4B versus encountering period in seconds. These results were obtained from an existing computer program, which is based on the theory described in References 5 and 6. Four different fins were chosen for the motion study. These were, respectively, the best aft fins judged from the transient characteristics—the 1.1 fin for SWATH 4A and the 1.0 fin for SWATH 4B—a fin greater than the best fin, a fin smaller than the best fin, and the best combination of the fore and aft fins.

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[6] Hadler, J. B. et al., "Ocean Catamaran Seakeeping Design Based Upon the Experience of USNS HAYES," Transactions Society of Naval Architects and Marine Engineers, Vol. 82 (1974).

TABLE 5 MOTION CRITERION AT 35 KNOTS

Fin Size		D_H	D_{HO}
Aft	Fore	Equation (15.5)	Equation (19)
SWATH 4A			
0.85	0	0.052	0.0982
1.0	0	0.045	0.108
1.1	0	0.040	0.104
1.2	0	0.033	0.0807
1.5	0	0.021	0.0304
1.6	0.4	0.0636	0.124
1.8	0.2	0.0325	0.0379
SWATH 4B			
0.5	0	0.0203	0.0430
0.67	0	0.0294	0.0584
1.0	0	0.0338	0.0840
1.2	0	0.0304	0.0901
1.5	0	0.0243	0.0559
2.0	0	0.0146	0.0213
1.6	0.4	0.0614	0.127
1.8	0.2	0.022	0.0671

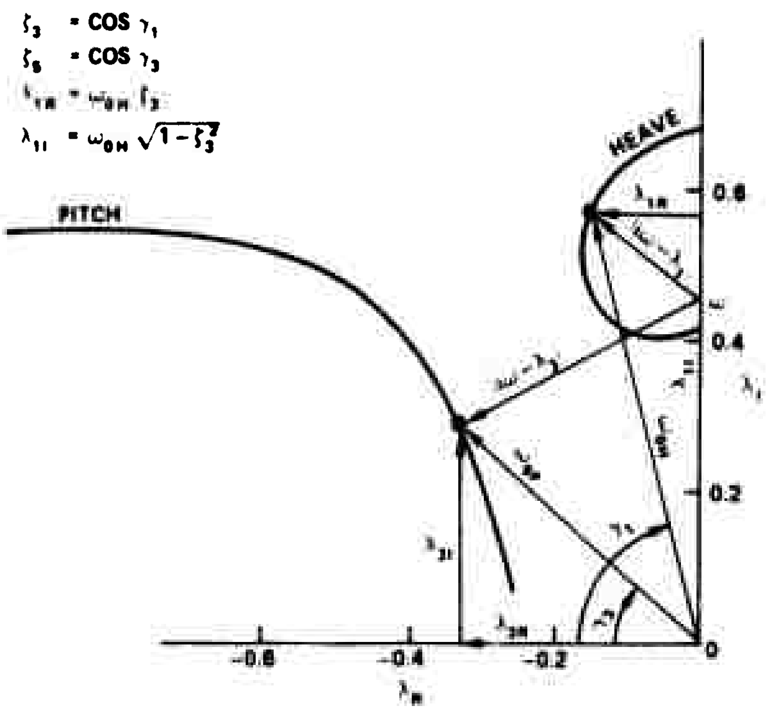

Figure 10 - Vector Diagram in Stability Root Plane

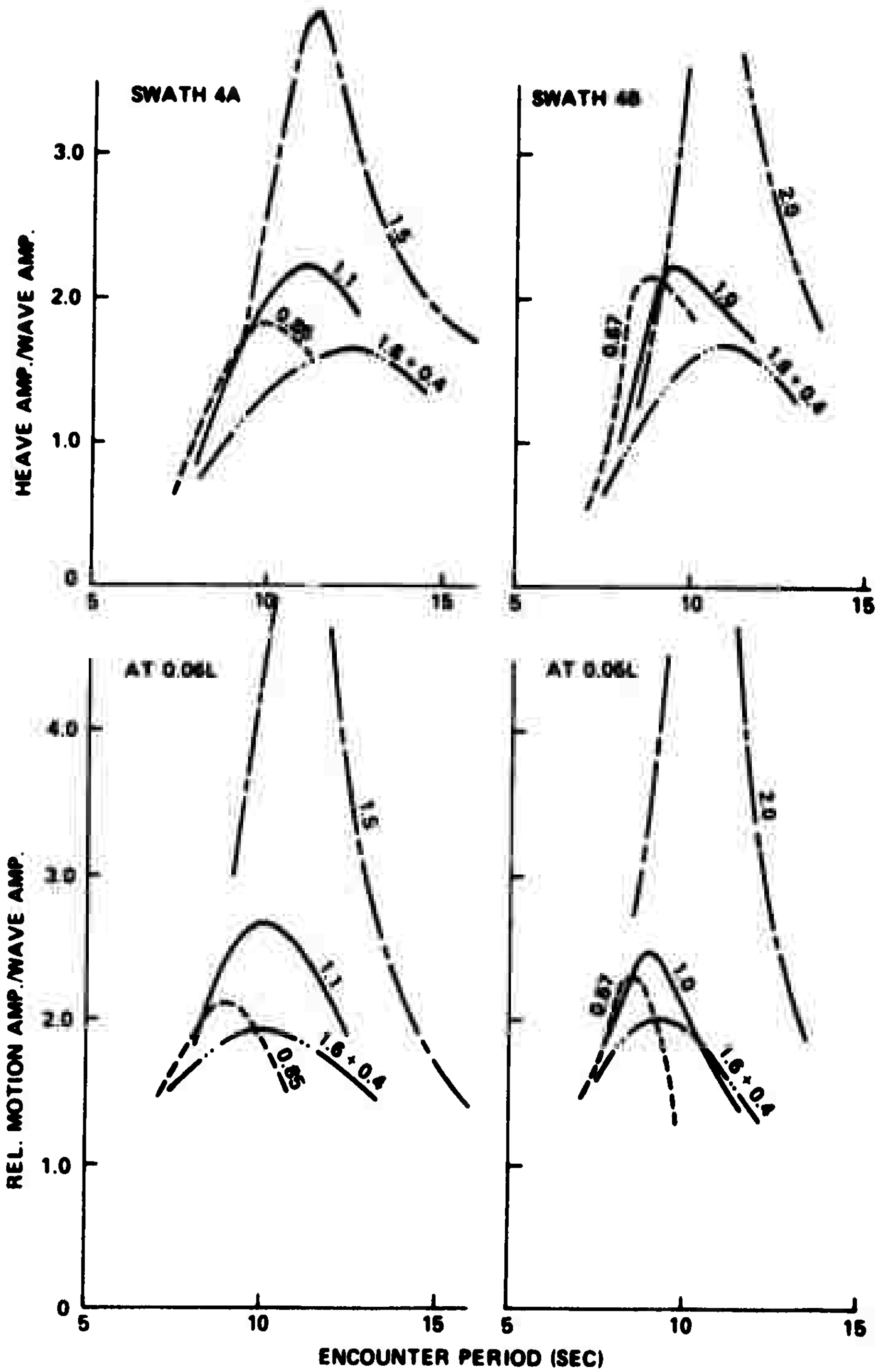

Figure 11 – Motion Response versus Encounter Period at 35 Knots

As can be seen from Figure 11, the periods at which the peak values of heave amplitude occur are very close to the heave natural periods given in Tables 3 and 4 as determined from the stability roots. The periods at which the peak values of the relative motion near the bow occur are slightly less than the corresponding periods for heave motion. This is understandable in view of Equation (16). The fins with greater heave damping ratio show a lesser degree of rise and fall of the motion amplitudes near the resonance, which may be expected. It is interesting to note that the configurations with the 1.5 and 2.0 fins, which were the largest shown, had the greatest motion at resonance. This was, of course, due to the fact that the heave damping ratios, as determined from the stability analysis, were only 0.12 and 0.07, respectively. The values were less than half the value determined for the best configurations; see Tables 3 and 4.

The effect of the wave-exciting term on the peak heave motion is apparent for the 0.67 fin for SWATH 4B. From Table 5, the fin which gives a maximum value for D_H can be found to be the 1.0 fin, whereas the 0.67 fin shows a slightly better response than the 1.0 fin as can be seen in Figure 11. However, in general, the frequency response of the ships with different fin configurations near the heave mode resonance is relatively well reflected by the values of D_H in Table 5. Although it is seen that for SWATH 4A, the maximum value of D_H is obtained for the 0.85 fin, it should be noted that the pitch mode is beginning to become less stable than for the larger fins. It can be seen from Figure 4 that the configuration with the 0.7 fin is on the verge of instability in pitch at 40 knots. In the selection of an optimum fin, on the basis of the present analysis, where many of the coefficients have been estimated, it would appear that a conservative approach should be used, and that a fin size yielding somewhat larger stability should be selected.

Within the context of the present analysis and under the consideration of pitch stability, motion characteristics in head waves, and the natural periods, it appears that for aft fin alone, the 1.0 to 1.2 fins for SWATH 4A, and the 0.7 to 1.2 fins for SWATH 4B are the best choices of those considered. For the fore and aft fin combination, the 0.4 fin forward and the 1.6 fin aft for both SWATH 4A and 4B appear to be the best choices.

CONCLUDING REMARKS

Suitable sizes for stationary stabilizing fins for SWATH's 4A and 4B were sought. The investigation revealed the following results.

1. Inception speed for instability is well predicted by Equation (17.3) derived from one of Routh's necessary conditions for stability.

2. Within the context of the present work it appears that suitable aft fins would be as follows:

	Number of Fins	Location	Chord ft	Span ft	Aspect Ratio
SWATH 4A	One for each hull	0.84 L at the inboard side of each hull	12.0~13.0	14.4~15.6	1.2
SWATH 4B	One for each hull	0.85 L at the inboard side of each hull	8.5~11.0	10.2~13.2	1.2

3. For a given hull with aft fins only, there seems to exist a certain size of fin which provides a maximum heave damping. As the size of fin is increased beyond a certain point, the stability and damping ratio of heave mode are decreased.

4. Combination of fore and aft fins demonstrates the possibility of improved stability and motion characteristics over aft fins alone; however, resistance may be increased due to increase in appendages. Further analysis is required to determine whether this would be worthwhile.

5. The larger fins introduced pronounced coupling effects between heave and pitch mode. In fact, doubling the optimum fin size resulted in an unstable heave mode.

6. Although SWATH 4B has a larger inception speed for instability due to her larger $\overline{GM}_{\ell}$, the overall transient characteristics of SWATH 4A with an optimum fin are judged better than those of SWATH 4B with an optimum fin.